The Marāgha Generation of Logicians

The forty years or so between the completion of Khūnajī's *Kashf al-asrār* (most probably by 634/1237) and Urmawī's *Bayān al-ḥaqq* (written in 675/1276) were extraordinarily prolific in terms of new logical works produced in the East. Most of these works were compiled after the Mongols sacked Baghdād in 656/1258. A closer look at their contents will disperse any prejudice that may remain against philosophical works written after the fall of the ʿAbbāsid capital. With the terms of the debate set by Razī and Khūnajī, four scholars were instrumental in shaping a truly Arabic logical tradition. Three of them were connected to the astronomical observatory at Marāgha near present-day Tabrīz: Athīr al-Dīn al-Abharī, Naṣīr al-Dīn al-Ṭūsī, and Najm al-Dīn al-Kātibī. Another scholar of the same generation, Sirāj al-Dīn al-Urmawī, was not connected to the observatory but played an important role in synthesizing the logical tradition in an advanced handbook.

Against widespread skepticism among scholars of Islamic studies, the work of these logicians shows that Arabic logic did not degenerate into scholastic school science. On the contrary, by the end of the 7th/13th century, they had reinvented the canon of Arabic logic to include controversial issues in textbooks and established a dialectical praxis that encouraged original research in logic. An important aspect of this development was the emerging discipline of formal disputation that shaped the critical engagement with Rāzī.

PROBING RĀZĪ: ATHĪR AL-DĪN AL-ABHARĪ AND NAṢĪR AL-DĪN AL-TŪSĪ

Athīr al-Dīn al-Abharī was a contemporary of Khūnajī's, probably about the same age, and it is not unlikely that they met. Going by his toponym, he was likely born

in the town of Abhar near Qazwīn northwest of Tehran.[1] Contrary to what Bar Hebraeus claims, he was probably not himself Rāzī's student, but studied with the latter's student Quṭb al-Dīn al-Miṣrī (d. 618/1221) in Nīshāpūr.[2] He further pursued studies in astronomy and dialectics (with the luminary Rukn al-Dīn al-ʿAmīdī [d. 615/1218]) in Samarqand, and later in Mosul.[3] Mosul was a vibrant center of learning at the time, and there he also studied Ptolemy's *Almagest* at the Badriyya madrasa with Kamāl al-Dīn Mūsā Ibn Yūnus (d. 639/1242), whose teaching fellow (*muʿīd*) he became.[4] Ibn Yūnus also taught Ṭūsī and Urmawī, and he reportedly read Fārābī and Avicenna with the Christian philosopher Theodore of Antioch.[5]

In 625/1227–1228, Abharī came to Irbil, where he settled a year later to teach at the Dār al-Ḥadīth. There, the biographer Ibn Khallikān (d. 681/1282) was among his students.[6] Information on his later life is scarce. A colophon written by one of his students suggests that he traveled or lived in Seljuk Anatolia at the same time as Khūnajī, returning to Mosul in 643/1245, and later went to Persia (*al-ʿajam*), where he died, probably in Shabistar near Marāgha.[7] The traditional date of his death, 19 Rabīʿ II, 663/February 8, 1265, is likely inaccurate, for it appears from manuscript evidence that he was no longer alive in 656/1258 when Ṭūsī wrote his *Taʿdīl al-miʿyār* (Recalibrating the Measure), a refutation of Abharī's *Tanzīl al-afkār* (The Revelation of Thoughts).[8]

In his overall philosophical outlook, Abharī largely followed Rāzī's revisionist course.[9] In logic, he was substantially influenced by the revisionist ideas of Khūnajī: the recent edition of Abharī's *Muntahā l-afkār fī ibānat al-asrār* (The Ultimate Thoughts in Explicating Secrets), a work that survives in two recensions, shows that the earlier recension lacks many of the idiosyncratic revisionist innovations from Khūnajī's *Kashf* that are however included in the later recension.[10] He was also somehow connected to Ṭūsī and may have had a hand in preparing the foundation of the astronomical observatory at Marāgha, even though it is doubtful that he ever worked there.[11]

Abharī's writings on astronomy and mathematics, as well as his connections to Frederick II of Hohenstaufen, have attracted the interest of scholars for a while now.[12] Yet his considerable output on logic remains largely unstudied and even unedited.[13] Next to his enormously influential tripartite presentation of philosophy titled *Hidāyat al-ḥikma* (The Guidance of Wisdom/Philosophy)—the logic part of which seems however to have been neglected by the later tradition—he is best known for his short introductory epistle on logic (*Īsāghūjī, eisagōgē*). It covers, in a highly succinct manner, all of the traditional logical corpus—not just the five *praedicabilia*, as had the texts by Porphyry and his epigones.[14]

Far from the esoteric appeal of the logic in the *Ishārāt*, it clearly was intended as an accessible "introduction" to all of logic that was to be read by budding logic students together with their teachers in a madrasa context. While the traditional

Eisagoge served as a propaedeutic to the Aristotelian *Organon*, Abharī's *Īsāghūjī* may be considered the first of several Arabic handbooks on logic whose purpose it was to introduce students to the principles of logic as was the state-of-the-art in the subject developed by the "later logicians (*al-muta'akhkhirūn*)."[15]

Abharī's lasting influence as a logician is largely due to the contribution he made to "school science" with his handbook, but it is his less-studied and more-advanced logical texts that help us better understand the emergence of logic as a research science in the second half of the 7th/13th century. Among the more influential of those texts are, first and foremost, the self-consciously revisionist tripartite presentations of philosophy, the *Kashf al-ḥaqā'iq fī taḥrīr al-daqā'iq* (Disclosing Truths in Revising Subtle Points) and the *Tanzīl al-afkār fī ta'dīl al-asrār* (The Revelation of Thoughts in Recalibrating Secrets).[16] On the latter Ṭūsī wrote a detailed refutation in the spirit of a more orthodox Avicennism.[17]

Of his other substantial logical works, the two recensions of his *Muntahā al-afkār* mentioned earlier appear to represent an intellectual turning point toward a more revisionist, specifically Khūnajīan position in logic.[18] It is noteworthy that Abharī wrote extensively on and reportedly taught dialectics (*jadal wa ādāb al-baḥth*) and juridical eristics (*'ilm al-khilāf*), both of which he had studied with the expert al-'Amīdī. For in the course of the century, we see an important increase in the influence of formalized rules of debate on logical commentaries.[19]

Ibn Khallikān reports that he himself studied *'ilm al-khilāf* with Abharī at the Dār al-Ḥadīth in Irbil, where he would probably have read Abharī's *Ta'līqa fī l-khilāf* (Notes on Juridical Eristics) and *al-Mughnī fī 'ilm al-jadal wa-ādāb al-baḥth* (Summa of Dialectic and Disputation Theory).[20] Abharī also taught Kātibī, whose formulations in the commentaries on Rāzī and Khūnajī are marked by strategies of formal disputation. But before we turn to Abharī's student, let us introduce his fellow scholar Ṭūsī.

Naṣīr al-Dīn al-Ṭūsī was born in 597/1201—about five years after Khūnajī and ten years before Rāzī's death—in the town of Ṭūs in Khurāsān.[21] Schooled by his father, a Twelver-Shi'ī jurist, he later—like Abharī—went on to study philosophy at the Niẓāmiyya madrasa in nearby Nīshāpūr with Rāzī's students Quṭb al-Dīn al-Miṣrī and Farīd al-Dīn Dāmād.[22] Around the time of the Mongol sacking of Nīshāpūr in 618/1221 he left for Baghdād and Mosul, where—again like Abharī—he studied with Ibn Yūnus.

He then spent about thirty years at Alamut, the Isma'īlī fortress of the "Assassins." After the Mongols captured Alamut in 654/1256, he was able to secure the patronage of Genghis Khān's grandson and future Īlkhān Hülegü, who in 657/1259—just a year after the fall of Baghdād—entrusted Ṭūsī with directing the astronomical observatory at Marāgha. Ṭūsī then spent over a decade at Marāgha, attracting numerous astronomers, philosophers, and mathematicians. He died in Īlkhānid Baghdād in 672/1274.

Even though Ṭūsī studied with the same teachers as Abharī, and perhaps read the *Ishārāt* under Abharī,[23] his resistance to Rāzī's revisionism developed into an intellectual program that went far beyond Abharī's critical attitude toward his predecessors. This phenomenon may at least be partly explained by sectarian affiliations, and, by implication, Ṭūsī's political agenda in his quest for patronage. However, even though he is much less of an obscure figure than Khūnajī or Abharī, Ṭūsī's role at the interstices of scholarship, academic administration, and politics remains difficult to reconstruct considering the tendentious nature of the sources.[24]

At any rate, it is beyond doubt that Ṭūsī's influence on Twelver-Shiʿism—and Islamic intellectual history more broadly—as the great synthesizer of Avicennan philosophy and Twelver-Shīʿī theology has been such that it is still felt today. Two works, the *Tajrīd al-manṭiq* (Extracted Points of Logic), an abridgement of the eight books of the *Organon* completed upon arrival at Hülegü's court in 656/1258, and the *Tajrīd al-iʿtiqād* (Extracted Points of the [Twelver-Shīʿī] Creed), were still being widely read along with the commentaries by his student al-ʿAllāma al-Ḥillī in Twelver-Shīʿī colleges until around 1950.[25]

Among Ṭūsī's other substantial works on logic is the *Asās al-iqtibās* (The Foundation of [Knowledge-]Acquisition), a Persian summa completed 642/1244–1245 dealing exclusively with logic and reverting to the organizing structure of the Aristotelian *Organon*, the departure from which Ibn Khaldūn had bemoaned.[26] Further, there is the slightly later *Ḥall mushkilāt al-Ishārāt* (Solving the Problems of the Pointers), an *Ishārāt* commentary completed in 644/1246–1247 while still in the Ismaʿīlī context of Alamut, refuting numerous interpretations and modifications Rāzī had presented in his own commentary. Finally, the *Taʿdīl al-miʿyār fī naqd Tanzīl al-afkār* (Recalibrating the Measure in Criticizing the Revelation of Thoughts) is a systematic criticism of Abharī's revisionist presentation of the three parts of philosophy, completed in 656/1258 (the same year as the *Tajrīd al-manṭiq*) probably already under the patronage of Hülegü.[27]

It is especially the *Taʿdīl* that, together with Abharī's *Kitāb al-shukūk* and Ṭūsī's *Ishārāt* commentary, provides an insight into the logical controversies in the lead-up to the foundation of the Marāgha observatory.[28]

The Beginning of Adjudication: Ṭūsī's Ishārāt *Commentary*

When Ṭūsī in 644/1246–1247 completed the *Ḥall mushkilāt al-Ishārāt*, the reception history of the *Ishārāt* changed significantly.[29] Even though he was not the first to write an *Ishārāt* commentary after Rāzī, Ṭūsī's commentary inaugurated a new era of adjudicative commentaries, *al-Muḥākamāt* (Adjudications), in which the relative merits and shortcomings of different *Ishārāt* commentators were being weighed.[30] This contributed to an increased scrutiny also of more peripheral issues like Rāzī's Repetition Argument. Here is Ṭūsī's criticism of Rāzī:

TEXT 50: NAṢĪR AL-DĪN AL-ṬŪSĪ, *ḤALL MUSHKILĀT AL-ISHĀRĀT* (FAYḌĪ),
241.3–15

The distinguished commentator [Rāzī] raised an objection against the Shaykh [Av-
icenna] by saying: "writer" demands a connection to something else all by itself,
because it is one of the derived names, so that his [Avicenna's] saying "but it really
should be said 'Zayd, he [is a] writer'" is not correct. Rather, it is only correct for
non-derived names. He may have been careless in this objection because the verb
only connects to its agent-noun when [the agent-noun] precedes it, but the agent-
noun does not [normally] precede the verb in Arabic. Thus, [the agent-noun] does
not connect by itself to a name that precedes it in any possible case, like with the
grammatical subject and others. Hence, [the agent-noun], in order to connect to its
like when it attaches to it needs another copula, i.e., not the one [the agent-noun]
contains in itself. How could this not be, when this is the case with the non-derived
name? If his words "Zayd [is a] writer" were exchanged for "Zayd writes" for ex-
ample, so that the predicate is the verb itself, then similarly it should really be said
"Zayd, he writes," because the connection of "writes" to "Zayd" preceding it is not
the [same as the] connection of the verb to its agent-noun that connects to it all by
itself, but rather it is the connection of the grammatical predicate to the grammati-
cal subject. The verb is here together with its agent-noun in the position of a simple
grammatical predicate connected to the grammatical subject by a copula that is not
the connection of the verb to its agent-noun.

Ṭūsī's criticism is reminiscent of Khūnajī's treatment in the *Kashf*—but not quite
the same. To recapitulate: Rāzī had argued that statement-words and derived
names have a semantic structure such that they include in their overall significa-
tion the signification of the nexus. Hence, propositions with statement-words or
derived names as predicates are binary by their own nature on the level of lan-
guage, so that mentioning a copula would result in superfluous repetition. Khūnajī
had attacked Rāzī for misrepresenting and unduly accusing Avicenna of incoher-
ence while he, Razī, was being incoherent himself.

   The crucial point for Khūnajī was that what needs to be signified in any expres-
sion of a proposition is the nexus to a determinate subject, and not only that to an
indeterminate subject (which is the one implicitly signified by verbs and derived
names). Khūnajī made a distinction between the signification of "*huwa*" as the
agent-noun (*ism fāʿil*) implicit in the verb and its signification as the copula,
the former being a noun and the latter an auxiliary.

   Ṭūsī nowhere mentions Khūnajī, but at its core his criticism of Rāzī's Repetition
Argument is the same. However, Ṭūsī's distinction between two kinds of connec-
tion, one a connection (*isnād*) between the verb (*fiʿl*) and its agent-noun (*fāʿil*)
that is implicitly signified by verbs, the other the connection (*isnād*) between the
grammatical subject (*mubtadaʾ*) and predicate (*khabar*), is slightly different, and
so is his argument for it.

Contrary to Indo-European grammar where word order tends to require that the subject be put first with the verb subsequently qualifying it, in Arabic verbal sentences always begin with a verb. The verb's inflection signifies an indeterminate agent that is only subsequently specified. Just as non-derived names do not connect by themselves, verbs also do not if the word order is reversed. For once a sentence starts with a noun, there is no expectation for the specification of an implied indeterminate agent. Ṭūsī claims that in such cases it will also be necessary to mention a copula in order to signify the connection of the grammatical predicate to its subject.

A fortiori, and against Rāzī, propositions with derived names in the predicate place will also require the copula to be mentioned in order to signify a nexus to a determinate subject, for even though "writer" (*kātib*), just like "writes" (*yaktub*), implies "he" (*huwa*), the "he" so implied is distinct from the "he" that is needed to signify the nexus of the predicate meaning to a determinate subject.

The Tanzīl *and the* Taʿdīl

Ṭūsī's refutation of Abharī's *Tanzīl al-afkār*, the *Taʿdīl al-miʿyār*, written more than a decade after the *Ishārāt* commentary, restates the same position. We do not know when exactly Abharī wrote the *Tanzīl*, but he was probably dead by the time Ṭūsī finished the *Taʿdīl*. It is noteworthy, especially given that the issue did not feature in the likely earlier *Kitāb al-shukūk*, that Abharī succinctly but explicitly presents the position of Khūnajī/Ṭūsī:

TEXT 51: NAṢĪR AL-DĪN AL-ṬŪSĪ, *TAʿDĪL AL-MIʿYĀR FĪ NAQD TANZĪL AL-AFKĀR* (MUḤAQQIQ & IZUTSU), 159.1–160.11

[Abharī's] words: If the predicate in a proposition is either a verb or a derived name, it signifies a nexus to some subject. If the copula is mentioned, it signifies a nexus to a determinate subject. If you say "Zayd writes," the utterance "he" is implied in the parts of the statement-word, but it is an agent-noun, and the copula signifies the nexus. What is signified by one is not what is signified by the other.

I [Ṭūsī] say: The verb, when it is the predicate, signifies the nexus to some subject when it is taken in isolation, which is what happens when we say for example "Zayd writes" [the grammatically proper word order in Arabic is reverse, i.e., "*yaktubu Zayd*"]. Here, the utterance "writes" signifies in its essence the nexus to some subject, which is then specified by the utterance "Zayd." As for what happens when we say for example "writes Zayd" [here the Arabic word order is the grammatically improper "*Zayd yaktub*"], there is no difference in the meaning mentioned between this and our saying "Zayd [is a] man," since both are connected by the [hidden] pronoun "he" on the level of meaning, so that what is implied is "Zayd, he writes." They only differ in that "writes" needs "he" yet another time, whereas "man" does not, and this is the "he" implied in the statement-word which is the agent pronoun—which is different from the copula, for it is a noun while the other is a particle. The grammarians call one the partitive and adjuvative copula, and the other a nominative pronoun. Hence, the derived name is analogous to the verb. When we say for example "[Is] Zayd a writer?" [the word order in Arabic is "A Writer Zayd?" with an interrogative particle

prefixed to "writer"] it connects by itself, and when we say "Zayd [he is a] writer" it connects by means of the implied utterance "he."

His words: When the copula in a proposition is a verb, the proposition is called incomplete ternary, because it does not signify a nexus to a determinate subject; it is only the atemporal copula that signifies that.

I say: In terms of meaning, the verb does not signify the nexus to a determinate subject essentially; it nonetheless signifies it accidentally, because its agent specifies that subject which the verb does not signify by specification, like when we say, for example, "Zayd is (*yakūnu*) a writer." Here, the utterance "is" signifies the necessity of its being connected to some subject, whereas the utterance "he"—which is implied in the verb and refers back to Zayd—specifies the subject connected to it. This proposition, on the level of meaning, signifies the same as that which is signified by "Zayd, he [is a] writer" (*Zayd huwa kātib*), with the additional signification of time because of the utterance "is" that is added to it.

In terms of the utterance, the proposition whose [copula] has the morphology of a verb is called defective ternary, and that whose [copula] is in the form of a noun is called complete ternary. One in which no copula is mentioned is called binary. The utterance alone, without considering hidden pronouns and implied [meanings], requires in one of the two [cases] the nexus to an indeterminate subject, and in the other the nexus to a determinate subject, and in the third, it does not signify a subject.

Ṭūsī does not disagree with Abharī here but seems to be fleshing out Abharī's position by way of his own argument. Again, he makes a clear distinction between the copulative and the pronominal "*huwa*." The signification of the latter is contained in verbs, that of the former is not—or at least not essentially. Interestingly, Ṭūsī here further elaborates the distinction between these two significations in terms of essential and accidental features of word-classes. Verbs essentially signify a nexus to an indeterminate subject—and so do derived names—by dint of the implied pronoun.

But, once in the context of a sentence, verbs and derived names may accidentally signify a determinate subject. As with the anaphoric use of indexicals, the signification of the implied pronoun, i.e., the indeterminate subject, may become specified by mentioning the subject if the implied pronoun refers back to that subject. In that sense, "*Zayd yakūnu kātib*" signifies the same proposition as that expressed by "*Zayd huwa kātib*," except that in the former case a tense is specified. That is because in "*yakūnu*" the pronominal "*huwa*" is implied, and once it refers back to the subject in the context of a sentence, it determines the subject and thus acts like the copulative "*huwa*."

Other Works by Abharī and Ṭūsī

Abharī's position on the issue seems to have been consistent across his other works. In his *Kashf al-ḥaqāʾiq* (completed before Ṭūsī wrote his *Ishārāt* commentary, but after his own revisionist turn of the *Muntahā*) Abharī had stated:

TEXT 52: ATHĪR AL-DĪN AL-ABHARĪ, *KASHF AL-ḤAQĀʾIQ FĪ TAḤRĪR AL-DAQĀʾIQ* (SARIOĞLU), 58.4–11

The predicate, if it is a statement-word or a derived name, may connect to the subject by itself, because [in that case] it signifies a nexus to some subject, yet it does not signify a nexus to a determinate subject. So when the copula is mentioned, the nexus to the subject is specified and hence there is no repetition.

But if you say that in "Zayd writes" (*Zayd yaktub*) the expression "he" is implied at the end of the statement-word in Arabic, so that if we mention the copula, there will be repetition—then we say we do not concede that. This is because what is implied at the end of the statement-word is an agent-noun and the other is a copula and there is no repetition.

As for Ṭūsī, it is worth noting that in his most substantial logical work (even though not nearly as influential as the *Tajrīd*), the Persian *Asās al-iqtibās*, he summarizes his discussion on the declarative statement and how the combination of simple expression works as follows:

TEXT 53: NAṢĪR AL-DĪN AL-ṬŪSĪ, *ASĀS AL-IQTIBĀS* (MUDARRIS RAḌAWĪ), 67.4–10

What is to be retained from this discussion is that the primary parts of any proposition are not more than two. These two together with the composition make three things, but not three parts. For the composition is not a part, but the nexus of one part of the proposition to the other. If the composition were a part, there would be a need for a new nexus. As we cannot possibly count the composition as a proper part, we will have to consider it a formal part and not a material part, whereas the other parts are material parts. Attention to this fine point is important, for the slightest negligence about these points leads to grave error.

This is clearly a reworking of the conceptual structure in Rāzī's chapter on the proposition in the *Mulakhkhaṣ* (additionally noting a version of Bradley's Regress), but it need not contradict his criticism of Rāzī's challenge.[31] In the *Tajrīd*, he simply states that every proposition consists of two parts, and that the copula may be omitted (even though in Persian it must be mentioned, he adds), depending on whether the proposition is binary or ternary.[32]

It appears that Abharī, and then Ṭūsī, who provided new arguments, both reacted in the same vein as Khūnajī to Rāzī's Repetition Argument. They both insisted that what is implied by statement-words and derived names is an agent-pronoun and hence distinct from the copula. The former signifies the nexus to an indeterminate subject, while only the latter signifies the nexus to a determinate subject. For both, mentioning the copula did not cause repetition.

According to Ṭūsī, however, statement-words and derived names that imply an agent-pronoun may be understood as containing a pronoun that refers back to a subject already mentioned, thereby specifying the nexus to a determinate subject. Abharī's student Kātibī not only applied his critical attitude to his teacher but went

back to the works of Rāzī and Khūnajī themselves to form his opinion on logical matters. He discussed his views in exchanges with Ṭūsī, who employed him as a professor at the Marāgha observatory.

NAJM AL-DĪN AL-KĀTIBĪ AT THE MARĀGHA OBSERVATORY

Najm al-Dīn ʿAlī b. ʿUmar al-Kātibī (Dabīrān) al-Qazwīnī was born in 600/1204 in the town of Qazwīn in modern-day Iran.[33] According to the Ottoman historiographer Kātib Çelebī (d. 1067/1657), Kātibī studied not only with Abharī, but also with Ṭūsī.[34] This is however not corroborated by earlier sources. At any rate, Kātibī and Ṭūsī knew each other and must have influenced each other, for Ṭūsī hired Kātibī along with three other philosophers when he set up the Marāgha observatory in 657/1259.[35]

It seems that Kātibī was teaching at Marāgha from 658/1260 until at least 670/1271–1272. Among his students were the polymath Quṭb al-Dīn al-Shīrāzī (d. 710/1311) and the famous Shīʿī scholar Ibn al-Muṭahhar al-Ḥillī (d. 726/1325), who were both also students of Ṭūsī's. According to El-Rouayheb, "al-Ḥillī described al-Kātibī as a Sunnī Shāfiʿī, and there is no reason to doubt this description, even though other early sources are silent concerning al-Kātibī's sectarian affiliation."[36] He died in 675/1276 and was buried in his hometown.

Kātibī was one of the most prolific and influential logicians in the Arabic tradition, and a pivotal figure in the burgeoning revisionist Avicennism in 7th/13th-century Arabic logic. He penned what was to become the arguably most influential logical text in the Arabic tradition: the short handbook dedicated to the Īlkhānid vizier Shams al-Dīn al-Juwaynī (*reg.* 661–83/1262–1284), whose patronage he enjoyed. Titled *al-Risāla al-Shamsiyya* (Epistle for Shams [al-Dīn]), it was simply known as the *Shamsiyya* and it elicited more commentaries than any other non-introductory logical work. It was used, together with some of the major commentaries, for teaching well into the 20th century.

In addition, Kātibī authored several holistic presentations of philosophy that begin with a part on logic, a number of independent works on logic, and many commentaries on logical works by others as well as on his own shorter works. He also engaged in written discussions with Ṭūsī (and others) on several issues. One exchange concerns the nature of the proposition.[37] Kātibī remains one of the most severely understudied logicians relative to his output and merit.

In the years that Kātibī was active at Marāgha (from 658/1260 until at least 670/1271–1272), he not only wrote some of his most important works, but also taught a significant number of students. Next to Shīrāzī and Ḥillī, several influential scholars are reported to have come to Marāgha to study with him. Fakhr al-Dīn Abū al-Fatḥ al-Qazwīnī al-Ḥakīm (the Philosopher) studied logic with Kātibī from 665/1266–1267 until dying prematurely two years later (his father was nicknamed

al-Athīrī and was probably Abharī's *famulus*).[38] Qawām al-Dīn Abū ʿAlī al-Yazīrī al-Ḥakīm also studied logic with Kātibī in 667/1267–1268.[39] Muḥyī al-Dīn Abū Faḍl al-Kūfī al-Baghdādī (d. 703/1303–1304) studied with Ṭūsī and Kātibī at Marāgha from the age of twenty-three (beginning in 670/1271–1272).[40] As for Shīrāzī, he arrived at Marāgha shortly after its foundation in 658/1259–1260 to study with Kātibī.[41] And Ḥillī came to study the works of Rāzī, Khūnajī, and Abharī together with Kātibī and Ṭūsī.[42]

The *Shamsiyya* is clearly a Marāgha text, because its dedicatee, Shams al-Dīn, became vizier in 661/1262, and it is likely that Kātibī used it for teaching there.[43] Another Marāgha text is Kātibī's commentary on Rāzī's *Mulakhkhaṣ*. Even though only completed in 671/1272–1273, he must have worked on it while at Marāgha. In the proem Kātibī says that he first commented on the logic, and some years later on the philosophical and theological parts, and then revised the logic before dedicating it to Shams al-Dīn.[44] For Kātibī's commentary on Khūnajī's *Kashf* we have no absolute dating, but given its length, he may have still worked on it at Marāgha.

These commentaries are not intended for beginners or intermediate students, if they were used for teaching at all. They betray a concern with what we would have to classify as "research science." In these texts, Kātibī quibbles with advanced and sometimes minute details of logic in a way that cannot in the main have been intended as mere exegetical work or as a running commentary for students. The fact that during his tenure at Marāgha Kātibī wrote both an accessible handbook clearly intended for intermediate teaching and substantial and critical advanced commentaries suggests that at Marāgha logic was conceived not merely as a school science, but also as a research science for its own sake. So let us look at these commentaries.

The Analysis of Simple Categorical Statements in Sharḥ Kashf al-asrār *(Commentary on the Disclosing of Secrets)*

Kātibī's commentary on Khūnajī's seminal summa *Kashf al-asrār* is a mammoth work, one of the longest works on formal logic ever written in Arabic.[45] It is not a typical lemmatic commentary. Instead of quoting and then treating the main text (*matn*) lemma by lemma, in this work the *matn* is incorporated in Kātibī's continuous prose. Besides explicating and frequently criticizing Khūnajī's text, Kātibī also completed what Khūnajī had expressly planned to include but left unfinished, adding sections on induction, analogy, and the matters of the syllogism (demonstration, dialectics, rhetoric, sophistry, and poetics).[46] The commentary of the chapter "On the Copula" is largely intended to be explicatory, not polemical. But two points are noteworthy.

First, even though Kātibī agrees with Khūnajī that the question whether in Arabic the copula needs to be mentioned is an issue for grammarians to resolve, he does present an argument in support of Rāzī's Repetition Argument. Kātibī thinks

that Khūnajī's criticism of Rāzī's Repetition Argument is fair granted that the copula "*huwa*" has a signification distinct from that of the agent-noun. The argument only works if the elided pronoun "*huwa*" implied by statement-words and derived names does in fact not signify a nexus to a determinate subject.

But, Kātibī points out in a way very similar to Ṭūsī's position, the implied agent-noun may well signify a nexus to a determinate subject once in the context of a sentence, because then it refers back to the subject and thereby makes the subject to which it signifies a nexus determinate. In that case, mentioning the copula should be considered a repetition (which Ṭūsī had denied):

TEXT 54: NAJM AL-DĪN AL-KĀTIBĪ AL-QAZWĪNĪ, *SHARḤ KASHF AL-ASRĀR* (MS ISTANBUL: SÜLEYMANIYE CARULLAH 1417), FOL. 58R13–18

[As for the copula] "he (*huwa*)": If we say for example "Zayd writes" or "Zayd [is a] writer," for them [the grammarians/logicians?] the predicate is in reality not the statement-word alone in the first case, nor the derived name alone in the second case. Rather, the predicate in the first case is the entirety of what is expressed by the statement-word. That means it includes the agent-noun implied after it. In the second case, it likewise is the entirety of what is expressed. That means it includes the name and the hidden pronoun contained in it.

Hence, for the statement-word and the derived name, even if nothing in them by itself signifies the nexus to a determinate subject, the entirety of what is expressed by them and what is implicit after them in terms of hidden pronouns that refer back to the preceding grammatical subject may still signify the nexus to a determinate subject. If this is so, then mentioning the copula another time will be a superfluous repetition.

Second, Kātibī is most vocal in his criticism on the passage where Khūnajī states that the nexus of the subject to the predicate must be conceptually distinct from the nexus of the predicate to the subject. While Abharī seems to have been the first to offer a substantial criticism of conversion rules, it appears that Kātibī was important for tying the issue of conversion to the debates on the nexus and the copula.[47] In Kātibī's words, Khūnajī had claimed that we know that the nexus of one term of a proposition to the other by subject-hood is not the same as its nexus by predicate-hood, because "if the two nexus were in fact one and the same, then there would be no distinction between what is understood from a proposition and what is understood from its converse," and this is obviously not the case (fol. 58r24–25).

In other words, if there were no such conceptual distinction, then "Humans are writers" would be the same as "Writers are humans." But this is not so, because—as Khūnajī argued—from a modal point of view, the propositions have different truth-conditions: writers are necessarily humans, but humans are not necessarily writers. Finding fault with the conditional, Kātibī criticizes the *modus tollens* argument:

Thus did the author [Khūnajī] present it, but there is room for discussion here. For the mentioned conditional [i.e., if the two nexus are . . .] is faulty.

[This is so] for the subject-term, because [the conditional] is only true [if both of the following are true]. [First,] what is understood from the original proposition is an expression of the nexus of the proposition's subject to its predicate by way of subject-hood, and[, second,] what is understood from [the original proposition's] converse is an expression of the nexus to it by predicate-hood—but this is not so! Rather, the first nexus is external to what the original proposition expresses, and the second is internal to what makes up the quiddity of the converse. It does not follow from the identity of two things, one of which is external to a quiddity and the other internal to what makes up another quiddity, the conceptual identity of both quiddities.

And for the predicate-term, [the conditional is faulty,] because it is only true [if both of the following are true]. [First,] what is understood from the original proposition is an expression of the nexus of its predicate to its subject by predicate-hood, and[, second,] what is understood from its converse is an expression of the nexus to it by subject-hood—but this is not so, either! From the identity of the two there follows no conceptual identity of quiddities. This is evident.

Kātibī does not disagree with the general idea that there should be a conceptual distinction between the relation the subject bears to its predicate and the one the predicate bears to its subject. But he faults Khūnajī with having failed to see that only one relation is internal to the quiddity of a proposition, and for consequently having committed a formal fallacy. There is no logical implication between the parts of the conditional on which the *modus tollens* argument depends:

> If the two nexus were in fact one and the same, (P)
> then there would be no distinction between what is understood from a
>   proposition and what is understood from its converse, (Q)
> but there is such a distinction. (¬Q)
> Therefore, the two nexus are not one and the same. (¬P)

For Kātibī, P does not imply Q to begin with—or it only does if we have a mistaken idea of what the properties of the two nexus are. The relation between "humans" and "writers" may well be one relation, no matter whether one is predicated of the other or *vice versa*, if we consider that only one aspect of this relation, i.e., "_being a predicate for_," is ever relevant for the proposition, or in Kātibī's words, "internal to the quiddity of the proposition."

If the two nexus are one and the same in this sense, this identity would not imply that we cannot distinguish between a proposition and its converse, because we can still make that distinction based on the different roles this nexus has. When "writers" bears the predicate-relation to "humans," the relation that "humans" bears to "writers" is irrelevant to the proposition. It only becomes relevant when

the proposition is converted, so that now "humans" bears the predicate-relation to "writers." None of this implies that a proposition and its converse are identical, for the same nexus is in one case internal to the quiddity of the proposition, and external in the case of conversion. Kātibī has more to say on this issue in his commentary on Rāzī's *Mulakhkhaṣ*.

The Analysis of Simple Categorical Statements in al-Munaṣṣaṣ fī Sharḥ al-Mulakhkhaṣ (The Precise Commentary on the Summary)

According to Kātibī, after having treated preliminary matters like the definition of "proposition" and its classification into different types, Rāzī now takes on the intrinsic properties (*arkān*) and immediate implications (*aḥkām*) of propositions themselves (fols. 31v31–32r16).[48] An intrinsic property of a proposition is what makes up its essence, i.e., its form and matter. Immediate implications are properties that are extrinsic to a proposition, but that are nonetheless determined by intrinsic properties, i.e., a proposition's contradictory, its *salva veritate* conversion, and so forth. The form of a proposition is its nexus, and its matter are subject and predicate.

The ensuing discussions deal, according to Kātibī, with issues to do with the form of propositions, first with questions about the nexus itself, then with questions about the utterance signifying it, i.e., the copula. Concerning the nexus, there are two issues. First, the claim that the nexus is a concept distinct from the concepts of subject and predicate, for which Rāzī offers two different arguments. Second, the claim that there are two conceptually distinct nexus in a proposition, for which Rāzī offers again two arguments. Kātibī first explains each of the arguments, and then advances his own criticism of each. I shall treat them in turn, and dwell a little on the last argument, about which Kātibī has the most serious misgivings.

The first argument for the claim that the nexus is a concept distinct from the concepts of subject and predicate is expressed, Kātibī thinks, by a conditional with two disjunctions:

> If the concept of the nexus were the same as the concept of the predicate, or
>> if the concept of the nexus were the same as the concept of the subject,
> then it would be impossible for us to conceive of the predicate separately, or
>> it would be impossible for us to conceive of the subject separately.

Both disjuncts of the consequent are false, for we are in fact able to conceive of both predicate and subject separately without at the same time conceiving of a nexus. If the conditional is evidently (*ẓāhiratayn*) true, as Rāzī must have thought, the falsity of the consequent implies the falsity of the antecedent. However, Kātibī adds, rather tersely: "there is room for discussion here, for we reject the possibility of conceiving the nexus that is between the two" (fol. 32r11). He does not elaborate this comment further, and we may only speculate what this criticism was supposed to amount to. He might have meant to directly question the conclusion

that the nexus is a concept distinct from the concepts of predicate and subject, by saying something like this:

> If the nexus is a distinct concept, it must be possible for us to conceive of it separately.
> But it is impossible to conceive of the nexus separately.
> Therefore, the nexus is not a distinct concept.

The second argument is given by Kātibī as follows: "The nexus which is between [subject and predicate] is posterior to both, for a nexus between two things is posterior to them, and what is posterior to something is necessarily distinct from it" (fol. 32r11–13). Here Kātibī takes issue with the first premise, i.e., that the nexus is posterior to subject and predicate. The way I read his counterargument is this. Take the totality of all nexus as a subject term of a proposition: "all nexus are posterior to their subject and predicate." Now think of the nexus to this subject, i.e., to "all nexus"—this nexus cannot be posterior to the subject, because it is also part of the subject. If this nexus is not posterior to the subject, it cannot be true that all nexus are posterior to subject and predicate (fol. 32r12).

He closes by saying:

TEXT 56: NAJM AL-DĪN AL-KĀTIBĪ AL-QAZWĪNĪ, *AL-MUNAṢṢAṢ FĪ SHARḤ AL-MULAKHKHAṢ* (MS MASHHAD: KITĀBKHĀNA-YI MARKAZĪ-YI ĀSTĀN-I QUDS-I RAḌAWĪ 1201), FOL. 32R13–16

Now that you have learned this, we say: as for the fact that every proposition inevitably needs to have a subject and a predicate, this is obvious; as for the nexus, it is internal to the quiddity of the [proposition], for if it were not, then anyone who conceived of the meaning of the subject and the meaning of the predicate without this nexus, would then conceive of the meaning of a categorical proposition—but it is obvious that this is not so.

The third argument Kātibī analyzes is the first Rāzī gives in support of the claim that in a proposition there are two conceptually distinct nexus. Kātibī presents the argument thus: "The nexus of the subject to the predicate is the nexus of the thing described to the description, and the nexus of the locus to that which occurs in it" (fol. 32r18). From this, says Kātibī, Rāzī intimates four syllogisms in the second figure that all produce the conclusion that the nexus of the subject to the predicate is not the same as the nexus of the predicate to the subject.

Kātibī objects:

TEXT 57: NAJM AL-DĪN AL-KĀTIBĪ AL-QAZWĪNĪ, *AL-MUNAṢṢAṢ FĪ SHARḤ AL-MULAKHKHAṢ* (MS MASHHAD: KITĀBKHĀNA-YI MARKAZĪ-YI ĀSTĀN-I QUDS-I RAḌAWĪ 1201), FOL. 32R23–26

There is room for discussion here, because we do not concede something in the premises mentioned in these syllogisms. For when we say "Every such-and-such is so-and-so," we do not mean that the first is the thing described and the second is a

description, and not that the first is a substrate and the second what inheres in it, even if in some kinds of propositions the first may be either something described or a substrate and the second either a description or what inheres in a substate. Rather, what we mean by it is that everything of which the first is true in actual fact, of that the second is also true. If this is so, then what he mentioned in terms of a proof for showing that the two nexus mentioned are distinct crumbles.

Rāzī's argument for a proposition's having two distinct nexus was based on the idea that there is a substantial logical distinction to be made between a description and the thing described by it. Kātibī counters that the logical form of a true proposition does not require any such distinction. Instead, what we mean when we say, for example, "All writers are humans" is that for all objects x of which it is true to say that they are a writer, it is also true to say that they are human. Whether or not *writing* is a description and *human* the thing described is simply irrelevant. Hence, Rāzī cannot build a proof on this distinction. It appears that Kātibī would here urge to treat both "human" and "writer" effectively as predicates, as we would do in modern logic. He does not, however, develop this idea further in the remainder of his discussion.

Whereas with regard to the first claim, i.e., that the nexus is a distinct concept, Kātibī's criticism was directed against the argument and not the claim itself, it seems that Kātibī was more seriously unhappy with the idea that in a proposition there are two conceptually distinct nexus. After refuting the first argument, Kātibī goes to some lengths to also refute the second. Kātibī understands the argument from distinct modalities as twofold, just like in his commentary on Khūnajī's *Kashf*. The first aspect is to say that if the nexus of the subject to the predicate were the same as the nexus of the predicate to the subject, then we could make no difference in the modal qualities of these relations. But the consequent is false, because, for example, in the proposition "Every writer is a human" the nexus of the subject to the predicate is necessary, whereas the nexus of the predicate to the subject is contingent.

The second aspect of the argument makes direct reference to conversion. Kātibī formulates it as follows: "To give an idea of what he means is to say that if the two nexus mentioned were identical, then propositions would retain their modalities in the converse. The consequent is false, as you will learn in the chapter on conversion, and hence the antecedent is rejected" (fols. 32r33–32v1). Kātibī has nothing more to say on this, but he strongly disagrees with Rāzī in the next lemma he quotes, where Rāzī states: "But the nexus which is a part of the quiddity of the proposition is that of the subject's essence being described by a predicate, whereas the other is necessarily external to it."[49] Kātibī appositely objects that the matter is more fittingly described as being the opposite: what matters, and what is part of the quiddity of a proposition, is not the nexus of the subject to the predicate, but the nexus of the predicate to the subject—though he does give Rāzī credit for having said as much in his *Ishārāt* commentary (10.11–11.4). He closes this section by saying:

In general, this is an issue that needs reflection. Inquire for yourself and seek the truth about it. My opinion is that the nexus of one of the two terms of the proposition to the other as being a subject for it is not the same as the nexus to it as being a predicate for it. If the two nexus were identical, then they would also have to be identical in their implications. But the consequent is false, because the nexus of the subject to the predicate as being a subject for it is external to the quiddity of both the original proposition and its converse. And its nexus to it as being a predicate is internal to the quiddity of the converse. The nexus of the predicate to the subject as being a predicate for it is internal to the quiddity of the original proposition and its converse. And the nexus to it as being a subject for it is external to the quiddity of both the original proposition and its converse together. The nexus of one of the terms of a proposition to the other as being a subject for it is thus not the nexus of the other to it as being a predicate for it. Hence, if these two nexus were identical, then there would be no difference between the subject of a proposition and its predicate inasmuch as they are subject and predicate. The consequent is obviously false. But one of these two nexus is the other *potentialiter*, and those cannot differ in quality or modality, because when "writer," for example, insofar as "human" is affirmed of it is necessary, then "human" is, insofar as it is affirmed of "writer," also necessary.

For Kātibī, there are four ways in which we can conceptualize the nexus as a relation between two terms. This is because you can convert any given proposition that consists of two terms. By exchanging subject and predicate you have two propositions, (2) and (4), in which the nexus may carry different modalities. While the nexus ($\leftarrow$) in (2) carries the modality of possibility ($\diamond$), because humans are only possibly writers, the nexus in (4) carries the modality of necessity ($\square$), because a writer is necessarily human. But you may also consider these two propositions in a different way, namely, by asking what modal relation the subject bears to the predicate. I distinguish these two types of nexus by writing ($\rightarrow_{\text{subjecthood}}$) and ($\leftarrow_{\text{predicatehood}}$). "Human" is necessarily a subject for "writer," but "writer" is only possibly a subject for "human."

$$(1)\ S(\text{human})\square(\rightarrow_{\text{subjecthood}})P(\text{writer}) \qquad (2)\ S(\text{human})\diamond(\leftarrow_{\text{predicatehood}})P(\text{writer})$$
$$(3)\ S(\text{writer})\diamond(\rightarrow_{\text{subjecthood}})P(\text{human}) \qquad (4)\ S(\text{writer})\square(\leftarrow_{\text{predicatehood}})P(\text{human})$$

Kātibī's position is that none of these nexus are in fact identical. First, he argues against Rāzī that the type of nexus in (1) and (3) is not part of the quiddity of these propositions, whereas the type of nexus in (2) and (4) is part of their quiddity. What we mean when we say "All writers are human" is not that *writing* is only possibly true of *human*, but rather that *human* is necessarily true of *writing*. Hence, the nexus in (1) is not identical to that in (2), and that in (3) not identical to that in (4). Second, Kātibī adds that while the nexus in (1) and (3) viz. (2) and (4),

respectively, do not necessarily have the same quality and modality, the nexus in (1) is potentially the nexus in (4), and that in (2) potentially that in (3), for in those pairs quality and modality are the same.

There follows a discussion on the copula that contains Kātibī's criticism of the Repetition Argument. He quotes Avicenna to the effect that, even though derived names and statement-words implicitly signify a nexus, they only signify a nexus to an indeterminate subject; but since a proposition requires a nexus to a determinate subject, Kātibī continues, this would mean that—if we follow Avicenna—in propositions in which the predicate is a derived name or a statement-word, a copula is still required to signify that nexus (fol. 32v23–25). But, in fact, in such propositions, it is not the derived name or statement-word alone that makes up the predicate, but also the agent-pronouns implied by them. Once in the context of a sentence, they do signify a nexus to a determinate subject, because they refer back to the subject already mentioned, in which case no copula is required (fol. 32v25–23).

As for Rāzī's argument itself, i.e., that, since the agent-pronoun is implied by derived names and statement-words, mentioning the copula would amount to repetition (e.g., "*Zayd huwa ʿālim huwa*" [Zayd, he is knowing he]), Kātibī does not concede that this is in fact a repetition, because that would only be the case if the second mention of "*huwa*" were not an agent-noun (fol. 32v31). But Rāzī did say it was an agent-noun, even though some grammarians call it an auxiliary (fol. 32v31–33). In any case, Kātibī says—just as he does in the commentary on Khūnajī's *Kashf*—that this is a question for grammarians to resolve; all that matters for the logician is that in some way or other the nexus to a determinate subject is signified (fols. 32v33–33r6).

SIRĀJ AL-DĪN AL-URMAWĪ AND THE NEW LOGIC HANDBOOKS

Sirāj al-Dīn Maḥmūd b. Abī Bakr al-Urmawī likely hailed from Urmia in the modern-day Iranian province of Azerbaijan.[50] He was born in 594/1198 and only twelve when Rāzī died. Hence, Bar Hebraeus's report of a teacher-student relationship is, again, likely false.[51] But he did study, like Abharī and Ṭūsī, with Ibn Yūnus, their fellow Shāfiʿī jurist, who was highly respected for his ability to explain Rāzī's texts.[52]

Little is known about Urmawī's early years. Later he migrated, like Khūnajī, first to Ayyūbid Egypt, where he enjoyed the patronage of al-Malik al-Kāmil and al-Malik al-Ṣāliḥ.[53] The latter sent him on a mission to the Hohenstaufen king of Sicily and Holy Roman emperor Frederick II.[54] According to his own testimony[55] in 655/1257 he moved, like Khūnajī, from Cairo to Seljuk Anatolia, where he spent the last decades of his life and was later appointed chief judge (*qāḍī*) of Konya. He died in Konya in 682/1283.

Urmawī came from the same historical region of Azerbaijan as Khūnajī. About the same age, later in their lives they were active at the same courts in Cairo. It is not unlikely that they met, but neither seems to mention the other in his writings. In terms of logical doctrine both authors are closely aligned in their revisionist ideas, and on grounds of the circumstantial evidence about the direction of influence, El-Rouayheb suggests that Urmawī was more likely a follower of Khūnajī's than the other way around, and that we might want to see "the logic part of the *Maṭāliʿ al-anwār* as [. . .] an abridgement of Khūnajī's *Kashf al-asrār*."[56]

At any rate, Urmawī's approach to the logical tradition was still critical overall. His most influential logical work was no doubt the *Maṭāliʿ al-anwār* (The Dawning of Lights), a more advanced handbook on logic and metaphysics (the metaphysics part fell out of use soon after Urmawī's death) significantly more detailed than Kātibī's *Shamsiyya*. Together with the lengthy commentary by Quṭb al-Dīn al-Rāzī al-Taḥtānī (d. 766/1365), it remained the standard handbook for advanced logical studies in the Eastern Islamic world well into the 19th century.[57]

Of the three handbooks written in the second half of the 7th/13th century—that is, Abharī's *Īsāghūjī*, Kātibī's *Shamsiyya*, and Urmawī's *Maṭāliʿ al-anwār*—the latter brought "school science" in logic closer to "research science." With it, many of the logical issues that had been raised in the course of the 7th/13th century became part of the advanced logic curriculum. These three logic handbooks institutionalized Arabic logic as a scientific discipline with a standard curriculum to be studied in the madrasa or outside of it. Logic had now not only completed the process of emancipation from the Aristotelian *Organon*, but also dissociated itself from the direct exegetical engagement with Avicenna.

With the *Īsāghūjī*, students had an easily accessible introductory textbook to the Arabic logic of the "later logicians." Kātibī's *Shamsiyya* was to become the standard intermediate textbook, and Urmawī's *Maṭāliʿ* the advanced work of reference for students who were interested in going beyond the expository textbooks. It was largely due to the commentaries together with which these texts were studied that they became so successful as teaching texts. By the early 10th/16th century it was virtually impossible that an accomplished scholar would have studied logic and not read any of these texts. The Ottoman scholar and judge Aḥmed Ṭāşköprüzāde (1495–1561), for example, reports that he studied the *Īsāghūjī* together with Kātī's commentary, the *Shamsiyya* with Quṭb al-Dīn al-Rāzī's commentary, and the the *Maṭāliʿ* together with Quṭb al-Dīn's commentary and al-Sayyid al-Sharīf al-Jurjānī's glosses.[58]

But not only was there now a readily available logic curriculum. The development of the genres of writing through which logic was presented was increasingly marked by the influence of the emerging science of formal disputation (*ādāb al-baḥth*), so that students who studied these handbooks together with advanced commentaries were at the same time trained to criticize logical arguments

themselves. In the centuries to come, the study of logic would, for many schol-
ars, culminate in the redaction of their own commentaries or glosses, which
were—arguably more often than not—not necessarily exegetical and intended for
teaching, but intended to advance the science of logic. They may often be seen as
contributions to original research.

Urmawī wrote widely on logic, and most of his works remain unedited.[59]
A good example to show how scholars in the latter half of the 7th/13th cen-
tury engaged with the history of a given logical problem and at the same time
contributed their own original thoughts by challenging their predecessors is
Urmawī's treatment of the copula in his summa *Bayān al-ḥaqq*. As the text gives
a succinct history of the problem of the copula and remains unpublished, it
is worth citing it in full:

TEXT 59: SIRĀJ AL-DĪN AL-URMAWĪ, *BAYĀN AL-ḤAQQ WA LISĀN AL-ṢIDQ*
(MS ISTANBUL: SÜLEYMANIYE ATIF EFENDI 1567), FOLS. 13R9–13V11

Second Section: On the Copula
    The categorical [proposition] is made up of three things: the subject, the predi-
cate, and the nexus by which one of them is connected to the other, in [the sense] that
it is it, or it is not it. If we conceive of both terms but do not conceive of the nexus as
we mentioned, then there is no conception of a proposition. Concerning this nexus
between them, from each of [the terms], it is only right that it be signified by an ex-
pression, and this expression is called the copula.
    [First inquiry:] If it is omitted in some languages or in certain contexts, then it
is just shorthand for what in principle must be expressed. It is only omitted on the
level of expression when it can be expected to be understood in the soul, either from
a [particular] language or in certain contexts. In that case the proposition is binary
on the level of expression. But it is ternary on the level of thought; if the [copula] is
expressed, then it is called ternary also on the level of expression.
    Second inquiry: The copula is no doubt one of the auxiliaries, but it may be in the
form of the hyparctic verbs mentioned earlier, in which case the copula is called a
"temporal copula" because of its signification of tense; or, it may be in the form of
a name, like any of the pronouns. Then the copula is—in the Arabic language—a
partitive or adjuvative copula, which [in logic] is called a "non-temporal copula."
Languages are different with regard to the use of the copula. In Greek it is necessary
to mention a temporal copula in all propositions, be they categorical or hypothetical.
In Arabic this is only necessary in conditionals like "If the sun is up, it is day." It is not
necessary in categoricals like "Zayd [is] in the house," when the proposition is binary.
When we say "Zayd was free," then [the proposition] is ternary, and the copula tem-
poral; when we say "Zayd, he [is] free," then the copula is atemporal. In Persian, it is
necessary that any proposition be ternary. The copula is either temporal as in "Zayd
was a writer" and "Zayd will be a writer," or atemporal, in which case it may be an
expression as in "Zayd is [*hast*] a writer," or a vocalization at the end of the predicate
as in "Zayd a writer [is]" [*Zayd dabīr-e*].[60]

Third [inquiry]: the Shaykh [Avicenna] said in the *Shifāʾ* that when the predicate is a verb or a derived name, it is not unlikely that it connects by itself to the subject, as it contains a nexus to the subject. Hence, the need of verbs and derived names for a copula [Avicenna continues] is not [the same as] the need of non-derived names [for it]. Then he said that indeed the verb and the derived name signify a nexus to a subject, but they do not signify a nexus to a determinate subject. Here, what is needed is something that connects the predicate to the subject, but the temporal copulae in Arabic do not signify a nexus to a determinate subject. Only atemporal copulae signify the nexus to a determinate subject. As they do in fact signify that, [Avicenna] distinguished three classes of propositions:

First: the complete ternary, which is the one in which a nexus to a determinate subject is signified, like the propositions in which there is a non-temporal copula.

Second: the incomplete ternary, which is the one in which an indeterminate subject is signified, like when the predicate contains a verb or derived name that includes, as mentioned, a temporal nexus.

Third: the binary—from this we know what he meant in the *Ishārāt* where he said that when we say "Zayd [is] a writer" it is necessary to [actually] say "Zayd, he [is] a writer," for by this the nexus is specified. He had explained that the nexus needs to be specified; but the need of verbs and derived names is not the [same as] the need of non-derived names, for there is nothing in the [latter] that signifies a nexus.

The Imām [al-Rāzī] falsely assumed that this was different from what [Avicenna] said in *al-Ḥikma al-mashriqiyya*, [namely] that the verb implicitly signifies the nexus to the subject. But I have ascertained that the two [passages] agree and there is no difference between them. The Imām said in his books that if the predicate is a verb or a derived name, then the proposition is in reality binary, because the nexus is signified by containment, and it is not permitted—on pain of repetition—to mention it separately. And if [the predicate] is a non-derived name [says Rāzī], then it is in reality ternary. On the basis of what you learned this is a weak argument.

In the commentary on the *Ishārāt* he ascertains that the verb only signifies the nexus to an indeterminate subject. Indeed, he said that if we say "Zayd writes," then the expression "he" (*huwa*) is hidden at the end of the verb—"concealed in it" as the Arabic grammarians say—and if we were to also place it in the middle, then we would have to say "Zayd, he writes, he." And because of this particle, the Imām believed that a repetition would follow.

I said: There only follows a repetition if each of the two expressions, the "he" in the middle and the "he" at the end, are copulae; but this is not so for the Arabic grammarians. Rather, the one at the end is an agent-noun and the one in the middle is a copula. Hence, they do not differ in that the one at the end is a name, but they differ with regard to the one in the middle—of which some [grammarians] say it is a name and others that it is an auxiliary. There appears in the Glorious Qurʾān a mention of the copula together with a predicate containing the nexus. This is in the word of the Exalted "And when You took me up, you were the Observer over them" [Q5:117]. Since ["Observer"] is being put in the accusative, the analysis of this on the part of the grammarians is that they say that the verb alone is not a predicate, but, together

with putting it into the accusative of the agent-noun and the verb, even if it does not signify a determinate subject on its own, they do so together.

But this is a linguistic inquiry—for the logician it is only necessary that he mentions whatever signifies a determinate subject, and if the verb and the derived name signify a determinate subject, then it is not necessary to mention the copula, and if not, then it is necessary to indicate it.

The temporal copula may be used for what is not temporal, like the words of Him Exalted: "He is [literally, was] compassionate and merciful," as well as for what has no specified time, like when we say "Every three is odd" and "Every four is even."

Fourth [inquiry]: The nexus of each of the terms to the other as being a subject for it is not [the same as] the nexus to it as being a predicate for it. Otherwise, a proposition would be the same as its converse, and the two would not imply each other. For they may differ in quality, like "Every human is an animal," but not "Every animal is a human," and in modality, like "Every human is possibly a writer," but "Every writer is necessarily a human." [. . .] The difference is [clear] in every proposition that does not convert. But every proposition has a converse that is not of their kind.

Further: The nexus of each of the two [terms] to the other by subject-hood is not [the same as] the nexus of the other to it by predicate-hood. Do they imply each other so that one of them is the other *potentialiter* and there is no difference in quality and modality?[61]

As for the quality, this is evident, because if A is a subject for B, it is impossible that B is not a predicate for A, no matter whether the subject-hood [of A] is affirmative or negative.

As for the modality, it has been said [by Kātibī] that if A insofar as B is affirmed of it is necessary, and B insofar as it is affirmed of A is necessary, then it is impossible that the two differ in modality.

There is room for discussion here: If the subject is more specific than the predicate, like "human" and "animal" for example, then "human" insofar as "animal" is affirmed of it is necessary, but "animal" insofar as "human" is affirmed of it is not necessary. This is with a view to their essences. As for the view to particular subjection and predication, like "This human is this animal," here it is necessary that each of the two nexus be necessary or non-necessary. It is impossible that there be a difference between them.

The Imām said: The nexus of one of them to the other by subject-hood is not [the same as] the nexus of the other to it by predicate-hood. It is because of this that the proposition does not preserve its modality in conversion. But this is weak. For in the converse subject-hood and predicate-hood differ.

Urmawī gives a detailed description of the history of the problem of predication up to his time. At several junctures of the story, Urmawī intervenes to criticize a position and propose an improvement. He also organizes the discussions on the copula into four inquiries: (i) the copula and whether it needs to be expressed, (ii) the copula and its grammatical description, (iii) the nexus and whether it is signified as part of the signification of verbs and derived names, (iv) the nexus and whether there are distinct nexus in a proposition.

It is noteworthy that Urmawī in his handbook reduces the inquiries to (ii) and (iv). However, even in the handbook, Urmawī criticizes Rāzī and includes a critical discussion of (iv). Rather than the *Bayān al-ḥaqq*, scholars in the later tradition often took the abridged version of the handbook (which is substantially the same) as their point of departure.[62]

CONCLUSION

The development of the discussions on the analysis of categorical propositions in the four decades between Khūnajī's *Kashf* (probably 634/1237) and Urmawī's *Bayān al-ḥaqq* (675/1276) may stand as a *pars pro toto* for the evolution of Arabic logic. The critical attitude fostered among Rāzī's disciples engendered discussions on logical issues big and small (in logic, there are no small distinctions, one might want to say) among leading scholars. All scholars in these chapters owe a great deal to Rāzī, and it was by no means Ṭūsī alone who became highly critical of the Rāzīan intellectual milieu in which he was nourished. Reading closely more of the unedited texts of this period will likely yield a picture more nuanced about not only the traditional Ṭūsī/Rāzī divide, but also the more recent distinction between orthodox and revisionist Avicennans.

Rāzī's challenge of Avicenna's position that a copula must be expressed in order to fully signify a complete proposition elicited critical reactions from all scholars discussed. Ṭūsī rejects it by giving arguments both from grammar and from semantic intuitions of natural language use. Ṭūsī is probably the first to substantially attack Rāzī on this point. Ṭusī sided with Abharī on the issue, as far as we can tell. But Abharī in his later work was highly critical of Rāzī's (and others') treatment of conversion rules for categorical propositions.

Kātibī seems to have been the first to offer substantial discussions of the subject-predicate nexus in relation to conversion and modality. All these discussions became, especially in the last two decades of the period under consideration, when Ṭūsī and Kātibī were active at Marāgha, both more sophisticated in advanced logical texts and synthesized (first and foremost by Urmawī, who was not connected to the observatory) into logic handbooks intended for students. The next generation of scholars was going to perpetuate this dual concern of teaching and research by producing some of the most influential commentaries to go with the new handbooks.